The Wiener process, also known as Brownian motion, is a continuous-time stochastic process that is widely used in various fields, including finance and physics. It is named after the mathematician Norbert Wiener and is characterized by the following properties:

1. **Continuous Paths:** The Wiener process has continuous paths, meaning that it is a continuous function of time.
2. **Gaussian Increments:** The increments of the process over disjoint time intervals are normally distributed.
3. **Independent Increments:** Increments over non-overlapping time intervals are independent.
4. **Stationary Increments:** The statistical properties of the process are time-invariant.

**Geometric Brownian Motion (GBM):**

Geometric Brownian Motion is a stochastic process that describes the evolution of an asset's price in financial modeling. It is used in the Black-Scholes-Merton model and has applications in option pricing. The GBM is defined by the following stochastic differential equation:

��(�)=��(�)��+��(�)��(�)*dS*(*t*)=*μS*(*t*)*dt*+*σS*(*t*)*dW*(*t*)

where:

* �(�)*S*(*t*) is the asset price at time �*t*,
* �*μ* is the drift (expected rate of return),
* �*σ* is the volatility,
* ��(�)*dW*(*t*) is the Wiener process increment.

**Derivations:**

1. **Wiener Process Derivation:**
   * The Wiener process is often introduced as the limit of a random walk. By considering a sequence of independent and identically distributed random variables with a scaling factor, the continuous-time limit results in the Wiener process.
2. **Geometric Brownian Motion Derivation:**
   * The GBM is derived from the Black-Scholes-Merton model, which is a partial differential equation describing the dynamics of financial markets. The solution to this equation leads to the GBM formula.

In this simulation, the Wiener process (left plot) and Geometric Brownian Motion (right plot) are generated using random increments. The resulting paths demonstrate the typical continuous and random behavior of these processes, respectively.